Chapter Eight: Geometry Mastery

This is a sample from
The PowerScore GRE Quantitative Reasoning Bible.

It comes from Chapter 8, which discusses
required knowledge from Geometry.
This specific section addresses
Geometric Solids.

POWERSCORE®
TEST PREPARATION

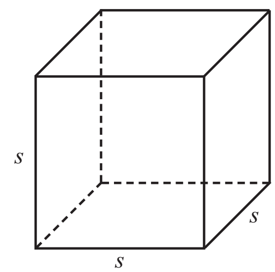
Geometric Solids

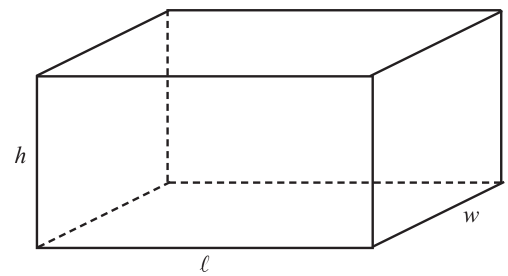
Frequency Guide: 3

A geometric solid is a three dimensional shape. The most commonly tested geometric solids are cubes, rectangular solids, and right circular cylinders. Other solids may appear, but their properties are not tested; they are usually just a foil for the properties of other shapes and solids.

Required Knowledge and Skill Set

1. Cubes and rectangular solids are six-sided "boxes."





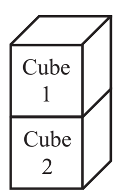
Volume = s^3
Surface area = $6s^2$
(s = the length of a side)

Volume = ℓwh
Surface area = sum of areas of six faces
Surface area = $2\ell w + 2hw + 2h\ell$

MEMORY MARKER: These formulas are essential for solving solid questions.

2. The surface area of a cube or a rectangular solid is the sum of the areas of the six faces.

3. When rectangular solids or cubes are stacked on top of each other, surface area pertains only to exposed portions of the solids. Unexposed portions of the figure must be subtracted from the total surface area of the individual components.



Unexposed Faces:
The bottom of Cube 1 and the top of Cube 2

If the side length of each cube is 3, the surface area of a freestanding cube would be 54:

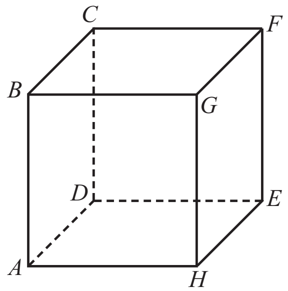
Total surface area of one cube = $6s^2 \rightarrow 6(3^2) \rightarrow 6(9) \rightarrow 54$

Because they are stacked, however, the area of one side must be discounted:

Surface area of Cube 1 = $5s^2 \rightarrow 5(3^2) \rightarrow 5(9) \rightarrow 45$
Surface area of Cube 2 = $5s^2 \rightarrow 5(3^2) \rightarrow 5(9) \rightarrow 45$

The total surface area of the stacked figure is the sum of the surface areas of Cube 1 and Cube 2: 45 + 45 = 90.

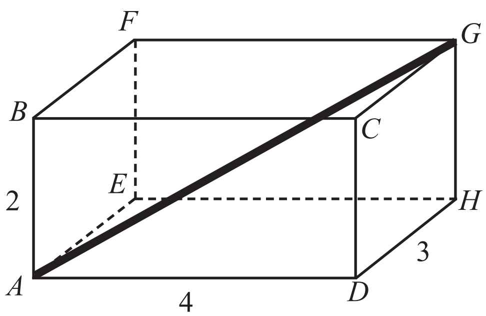
4. There are 8 vertices in a cube or rectangular solid. The farthest distance from a single vertex is its diagonal vertex:



$AB = AD = AH$
$AC = AE = AG$
$AF > AB$
$AF > AC$

5. To find the length of a diagonal in a cube or rectangular solid, use the following formula:

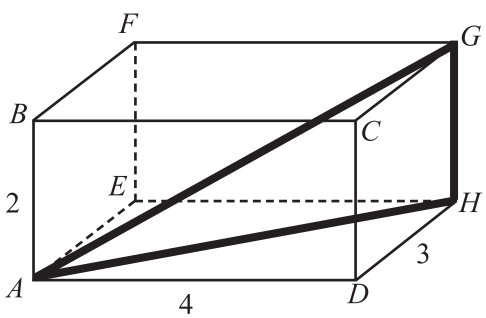
Length of a diagonal in a rectangular solid = $\sqrt{l^2 + w^2 + h^2}$



$$\text{Diagonal} = \sqrt{l^2 + w^2 + h^2}$$
$$= \sqrt{4^2 + 3^2 + 2^2}$$
$$= \sqrt{16 + 9 + 4}$$
$$= \sqrt{29}$$

MEMORY MARKER: The formula for the length of a diagonal in a rectangular solid should be memorized. Finding this distance using triangles can be a lengthy process.

If you forget the formula, use hidden right triangles to determine the length of the diagonal:



Length of AH:
$a^2 + b^2 = c^2$
$AD^2 + DH^2 = AH^2$
$4^2 + 3^2 = AH^2$
$16 + 9 = AH^2$
$25 = AH^2$
$5 = AH$

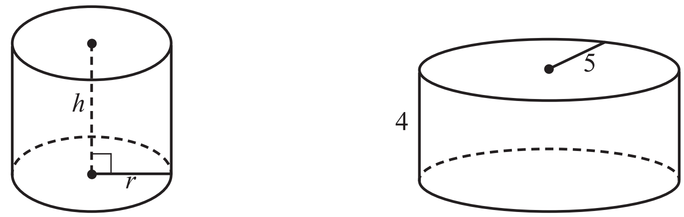
Length of AG:
$a^2 + b^2 = c^2$
$AH^2 + GH^2 = AG^2$
$5^2 + 2^2 = AG^2$
$25 + 4 = AG^2$
$29 = AG^2$
$\sqrt{29} = AG$

Memorizing the formula is more efficient, but hidden triangles can bail you out if you fail to recall the formula.

8

Can you draw a rectangular solid and right circular cylinder? A basic skill like sketching a cube or a cylinder can be invaluable on test day. Practice drawing each one in the space below.

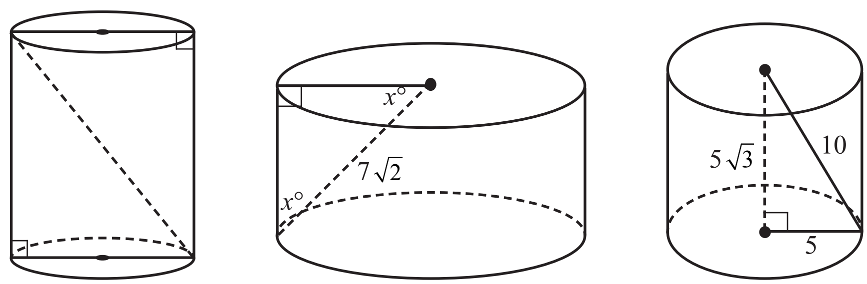
6. The GRE frequently tests right circular cylinders, which are geometric solids with equal circular bases joined by curved surfaces that form right angles at the circular bases.



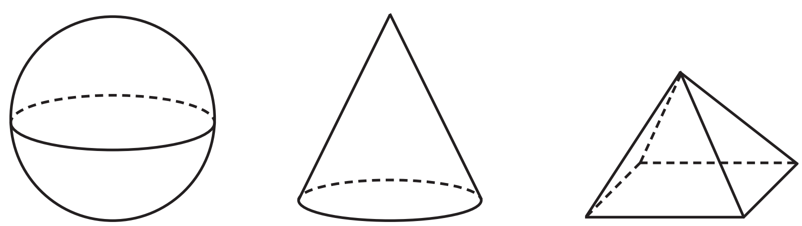
Volume = $\pi r^2 h$

Volume = $\pi r^2 h$
Volume = $\pi(5^2)(4)$
Volume = 100π

7. Right circular cylinders are often used to hide right triangles::



8. You may be asked to work with spheres, cones, or pyramids. You do not need to know any properties of these solids, but as always, you should be on the lookout for hidden triangles or other polygons.



Sphere Cone Pyramid

Note that the center of the sphere is a circle, the base of the cone is a circle, and the base of the pyramid is either a square or a rectangle. If these figures are used on the GRE, you will likely be asked to evaluate some part of the base using the formulas for these previously covered polygons.

8

Application on the GRE

The majority of geometric solid questions involve the volume of the figure. The easiest questions will simply ask for the volume of a cube, rectangular solid, or right circular cylinder:

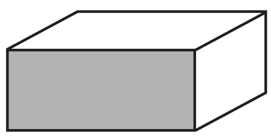
> A right circular cylinder has a height of 4 and a base with a radius of 3. What is the volume of the figure?
>
> (A) 12π
> (B) 24π
> (C) 36π
> (D) 48π
> (E) 144π

Remember to ANALYZE the answer choices! The only formulas that involve π are formulas for circles, sectors or arcs, and cylinders.

To solve, plug the values into the formula:

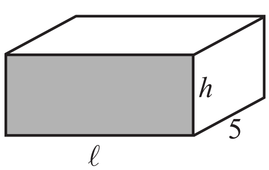
$$\text{Volume} = \pi r^2 h \quad \rightarrow \quad \pi(3^2)(4) \quad \rightarrow \quad \pi(9)(4) \quad \rightarrow \quad 36\pi$$

Volume questions involving cubes and rectangular solids are often worked "backwards," much like the circle questions that give the area and require the circumference. The volume of the solid is given, but the area of a single face is requested. Consider the following question:



> In the figure above, the rectangular solid has a depth of 5 centimeters and a volume of 150 cubic centimeters. What is the area, in square centimeters, of the shaded face?
>
> [ ]

DIAGRAM the question:



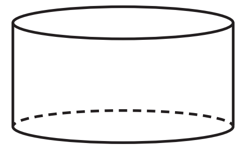
$\text{Volume} = \ell wh$
$150 = \ell(5)h$
$30 = \ell h$

Confidence Quotation
"Skill and confidence are an unconquered army."
-George Herbert, Welsh poet and advisor to King James I

The area of a rectangle is the length times the width. Because this rectangle is part of a rectangular solid, the area of the shaded face is the length times the height. Therefore, the area is 30.

Knowing that there are only two types of questions about right circular cylinders makes these questions easier to solve. If the text of the question does not use the word "volume," look for hidden triangles.

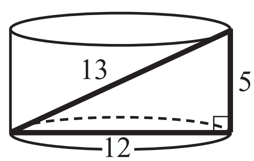
Questions about right circular cylinders fall into two categories: (1) questions involving volume and (2) questions involving triangles. The volume questions may be like the one we discussed on the previous page, where the volume must be determined, or they may give the volume and ask for the height of the cylinder or radius of the base. Right circular cylinder questions involving triangles usually have a Hard difficulty level, but they are easy if you remember to use the Pythagorean Theorem:



In the figure above, a cup used to hold drinking straws is a right circular cylinder with a height of 5 inches. The base of the cup has a radius of 6 inches. Of the following straw lengths, which is the longest one that can fit entirely in the can?

Ⓐ 11.2 inches
Ⓑ 12.8 inches
Ⓒ 13.4 inches
Ⓓ 14.6 inches
Ⓔ 15.0 inches

The longest straw that would fit in the cup would stretch from opposite points on the two bases. DIAGRAM the question to discover the triangle:



$a^2 + b^2 = c^2$
$5^2 + 12^2 = c^2$
$25 + 144 = c^2$
$169 = c^2$
$13 = c$

You should have immediately recognized the Pythagorean Triple, 5:12:13. If you did not, you can use the Pythagorean Theorem to find the distance between opposite points on the bases.

The longest straw that could fit in the cup is 13 inches. Therefore, the longest in the list of answer choices that would fit is answer choice (B), 12.8 inches. The straw in choice (C), 13.4 inches, is 0.4 inches too long.

If you encounter a question with a right circular cylinder, first assess whether it involves volume (these are usually medium questions). If the question does not use volume, find the hidden triangle (these are usually hard questions).

8

Similarly, easy and medium questions about cubes and rectangular solids tend to deal with volume. The more difficult questions address surface area. Some of these questions ask you to find the surface area of a stacked figure, as demonstrated in the Required Knowledge and Skill Set. Others are word problems that involve surface area and volume:

> In math class, students are given small plastic cubes. Each small plastic cube has a surface area of 24 square inches. If the teacher gives the students a large cube with a surface area of 864 square inches, how many small cubes can fit into the large cube?

Begin with the information you are given: surface area. Find the length of the sides of the cubes using the surface area formula.

Surface area of small cube = $6s^2$
$24 = 6s^2 \rightarrow 4 = s^2 \rightarrow 2 = s$

Surface area of large cube = $6s^2$
$864 = 6s^2 \rightarrow 144 = s^2 \rightarrow 12 = s$

Think of surface area as the amount of wrapping paper needed to cover a box.

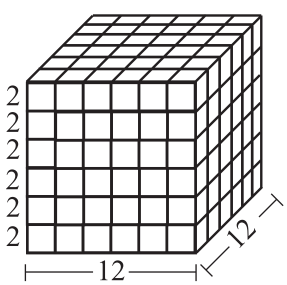
Think of volume as the amount of water needed to fill an aquarium.

Now use the side length to find the volume of each sized cube:

Volume of small cube = $s^3 \rightarrow 2^3 \rightarrow 8$

Volume of large cube = $s^3 \rightarrow 12^3 \rightarrow 1728$

If the volume of the large cube is 1728, how many cubes of volume 8 can it hold? Imagine the large cube filled with small cubes:

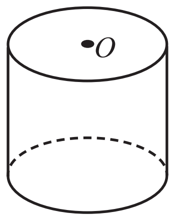


$1728 \div 8 = 216$

The large cube can hold 216 small cubes.

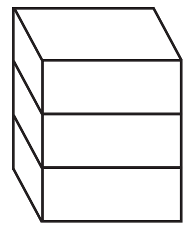
Geometric Solids Problem Set

Solve the following multiple-choice, numeric entry, or quantitative comparison questions following the directions for each question type as explained in Chapter 2. Answers begin on page 356.



1. In the figure above, a cylinder has a height of 4 and a diameter of 6. If point *O* is the center of the top circular base, what is the distance from *O* to any point on the edge of the bottom base?

   Ⓐ 2
   Ⓑ $\sqrt{5}$
   Ⓒ 5
   Ⓓ $2\sqrt{13}$
   Ⓔ 10



2. In the figure above, each of the three individual boxes has a depth of 3 inches, length of 5 inches, and height of 2 inches. When the three boxes are stacked together as shown, what is the surface area, in inches, of the resulting figure?

   Ⓐ 60
   Ⓑ 90
   Ⓒ 96
   Ⓓ 126
   Ⓔ 186

8

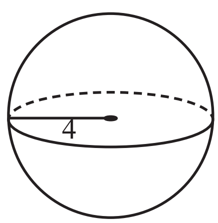
Geometric Solids Problem Set

Solve the following multiple-choice, numeric entry, or quantitative comparison questions following the directions for each question type as explained in Chapter 2. Answers begin on page 374

3. The volume of a right circular cylinder is 200π and the base of the cylinder has a circumference of 10π. A cube has side length of 5 units.

Quantity A	Quantity B
The height of the cylinder	The diagonal of the cube

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.



4. A sports equipment manufacturer created a spherical wooden ball that has a radius of 4 centimeters. The manufacturer wants to place the ball into the smallest box possible. If the box is a cube, what is the volume, in cubic centimeters, of the smallest box that will contain the entire ball?

[ ]

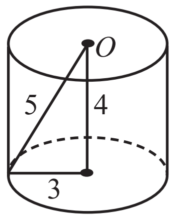
8

Geometry Mastery Answer Key

Geometric Solids Problem Set Answer Key—page 354

1. (C) Hard

DIAGRAM the question to see the right triangle:

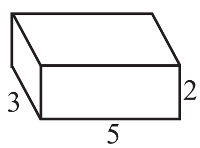


You should immediately recognize the Pythagorean Triple, 3:4:5. If not, use the Pythagorean Theorem:

$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$5 = c$

2. (D) Hard

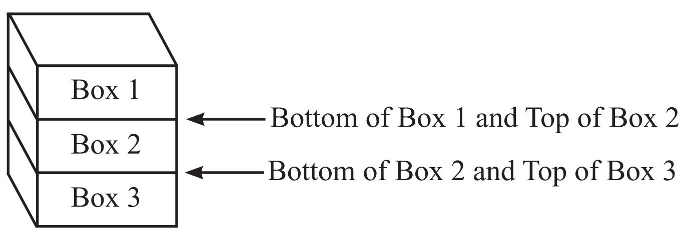
Find the surface area of the three individual boxes as if they were floating in space and not stacked together:



Surface area = $2\ell w + 2\ell h + 2wh$
Surface area = 2(5)(3) + 2(5)(2) + 2(3)(2)
Surface area = 30 + 20 + 12 = 62

Surface area of 3 boxes = 62 × 3 = 186

Now identify the unexposed faces:



There are 4 unexposed faces, and they all have the same area (length × width):

Area of unexposed face = ℓw = (5)(3) = 15

Area of 4 unexposed faces = 4(15) = 60

Now subtract the area of the unexposed faces from the surface area of three floating boxes:

186 – 60 = 126 The correct answer is (D).

3. (B) Hard

Quantity A is a classic "work backwards" GRE question. Use the circumference to find the radius:

Circumference = $2\pi r$
Circumference = 10π

$2\pi r = 10\pi \rightarrow 2r = 10 \rightarrow r = 5$

The only formula that uses the height of a cylinder is the volume formula. Use the radius and the volume to find the height:

Volume = $\pi r^2 h$
Volume = 200π Radius = 5

$\pi(5^2)h = 200\pi \rightarrow 25h = 200 \rightarrow h = 8$

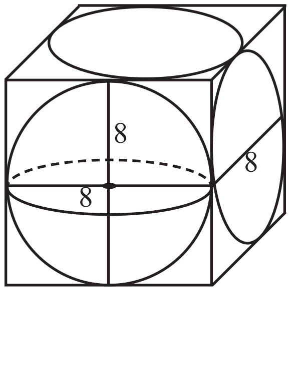
Now use the formula for the length of a cube's diagonal to find the value of Quantity B:

$\sqrt{l^2 + w^2 + h^2}$
$\sqrt{5^2 + 5^2 + 5^2}$
$\sqrt{25 + 25 + 25}$
$\sqrt{75}$
8.66025

Quantity B is greater than Quantity A.

4. 512 Hard

DIAGRAM the question:



The length, width, and height of the cube are all 8 centimeters.

Volume = $s^3 \rightarrow 8^3 \rightarrow$ 512 cubic centimeters